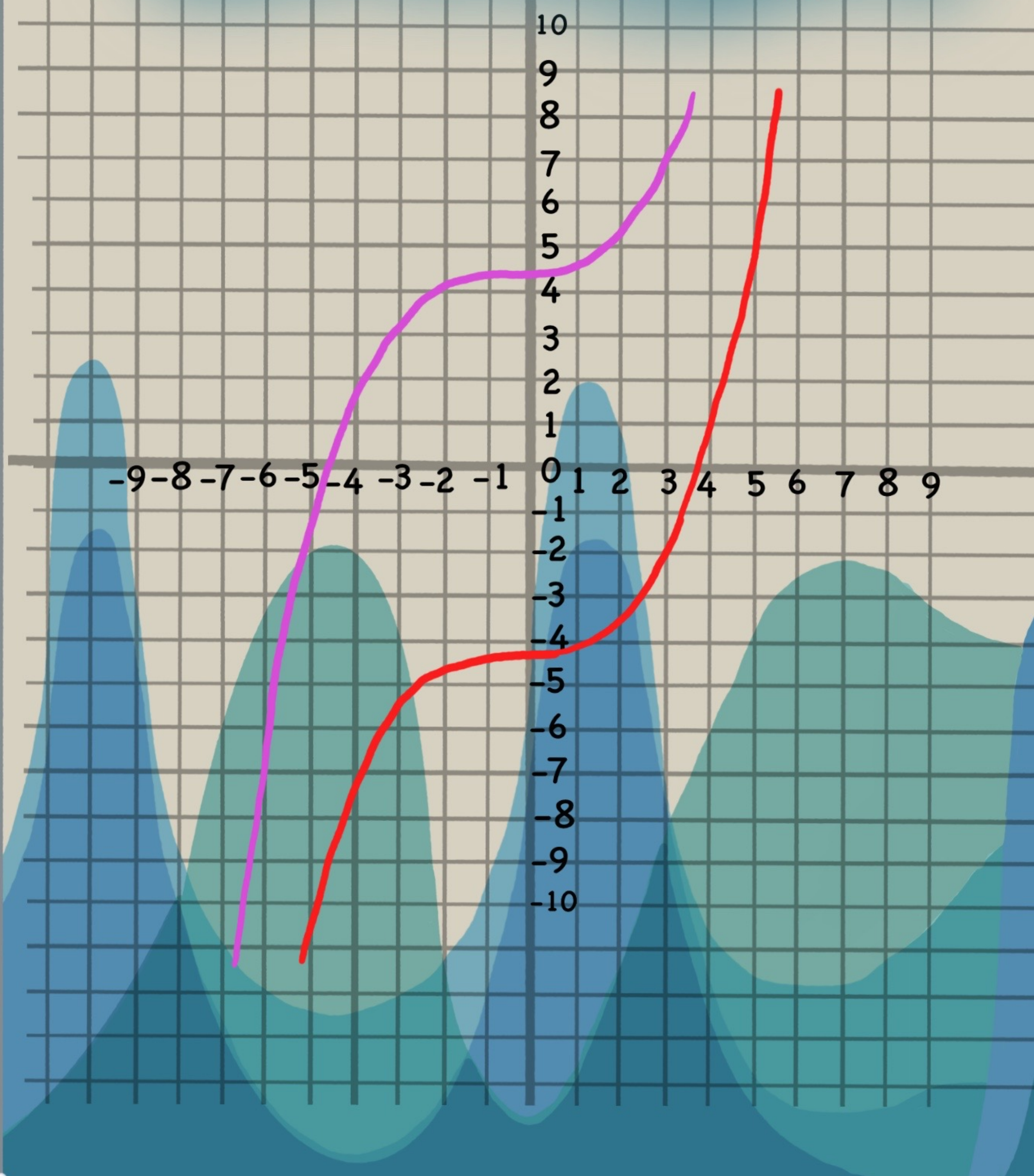
The Maths Society

Curve graph

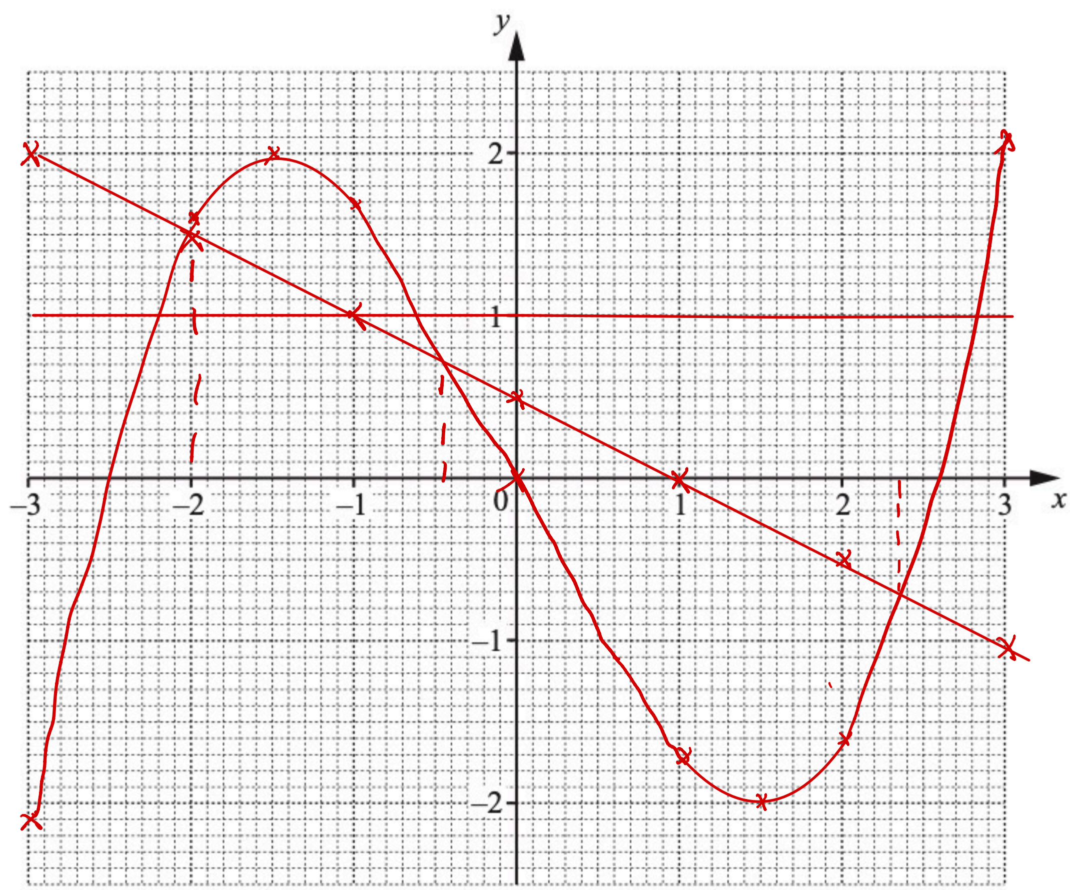


5 The table shows some values for $y = \frac{3}{10}x^3 - 2x$ for $-3 \leqslant x \leqslant 3$.

x	-3	-2	-1.5	-1	0	1	1.5	2	3
y	-2.1	1.6	2.0	1.7	0	-1.7	-2.0	-1.6	2.1

(a) Complete the table. [3]

(b) On the grid, draw the graph of $y = \frac{3}{10}x^3 - 2x$ for $-3 \leqslant x \leqslant 3$.



[4]

$\frac{1}{2} - \frac{1}{2}x$

(c) On the grid opposite, draw a suitable straight line to solve the equation $\frac{3}{10}x^3 - 2x = \frac{1}{2}(1-x)$ for $-3 \leqslant x \leqslant 3$.

$y = \frac{1}{2} - \frac{1}{2}x$

$x = $ ......-2...... or $x = $ ......-0.45...... or $x = $ ......2.35...... [4]

(d) For $-3 \leqslant x \leqslant 3$, the equation $\frac{3}{10}x^3 - 2x = 1$ has n solutions.

Write down the value of n.

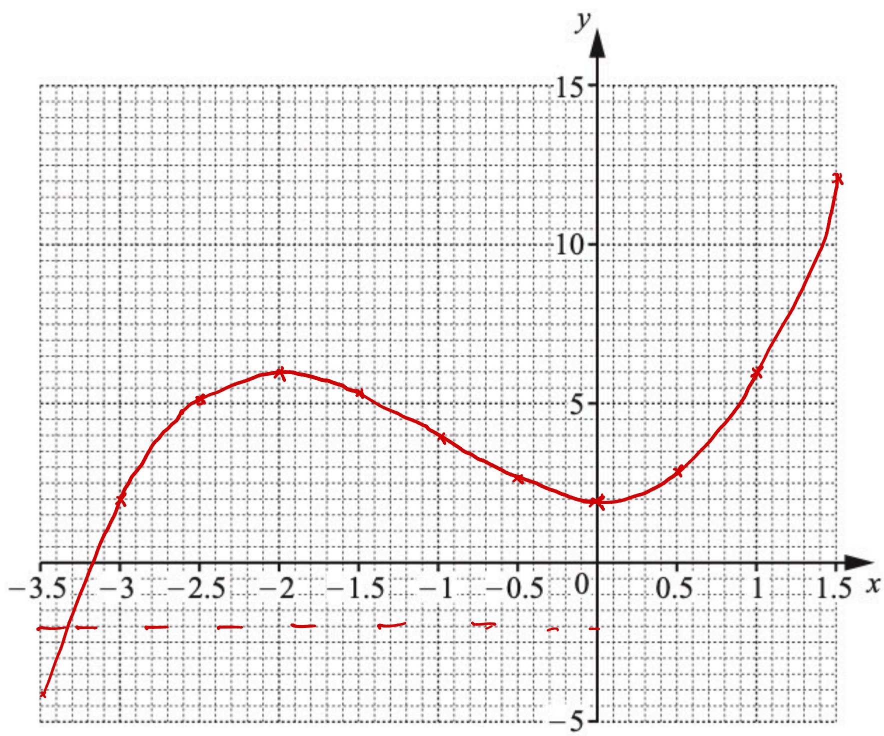
$n = $ ......3...... [1]

2 The table shows some values for $y = x^3 + 3x^2 + 2$.

x	-3.5	-3	-2.5	-2	-1.5	-1	-0.5	0	0.5	1	1.5
y	-4.1	2	5.1	6	5.4	4	2.6	2	2.9	6	12.1

(a) Complete the table. [3]

(b) On the grid, draw the graph of $y = x^3 + 3x^2 + 2$ for $-3.5 \leqslant x \leqslant 1.5$.



[4]

(c) Use your graph to solve the equation $x^3 + 3x^2 + 2 = 0$ for $-3.5 \leqslant x \leqslant 1.5$.

$x =$ -3.18 .... [1]

(d) By drawing a suitable straight line, solve the equation $x^3 + 3x^2 + 2x + 2 = 0$ for $-3.5 \leqslant x \leqslant 1.5$.

$x =$ -3.35 .... [2]

(e) For $-3.5 \leqslant x \leqslant 1.5$, the equation $x^3 + 3x^2 + 2 = k$ has three solutions and k is an integer.

Write down a possible value of k.

$k =$ 4 .... [1]

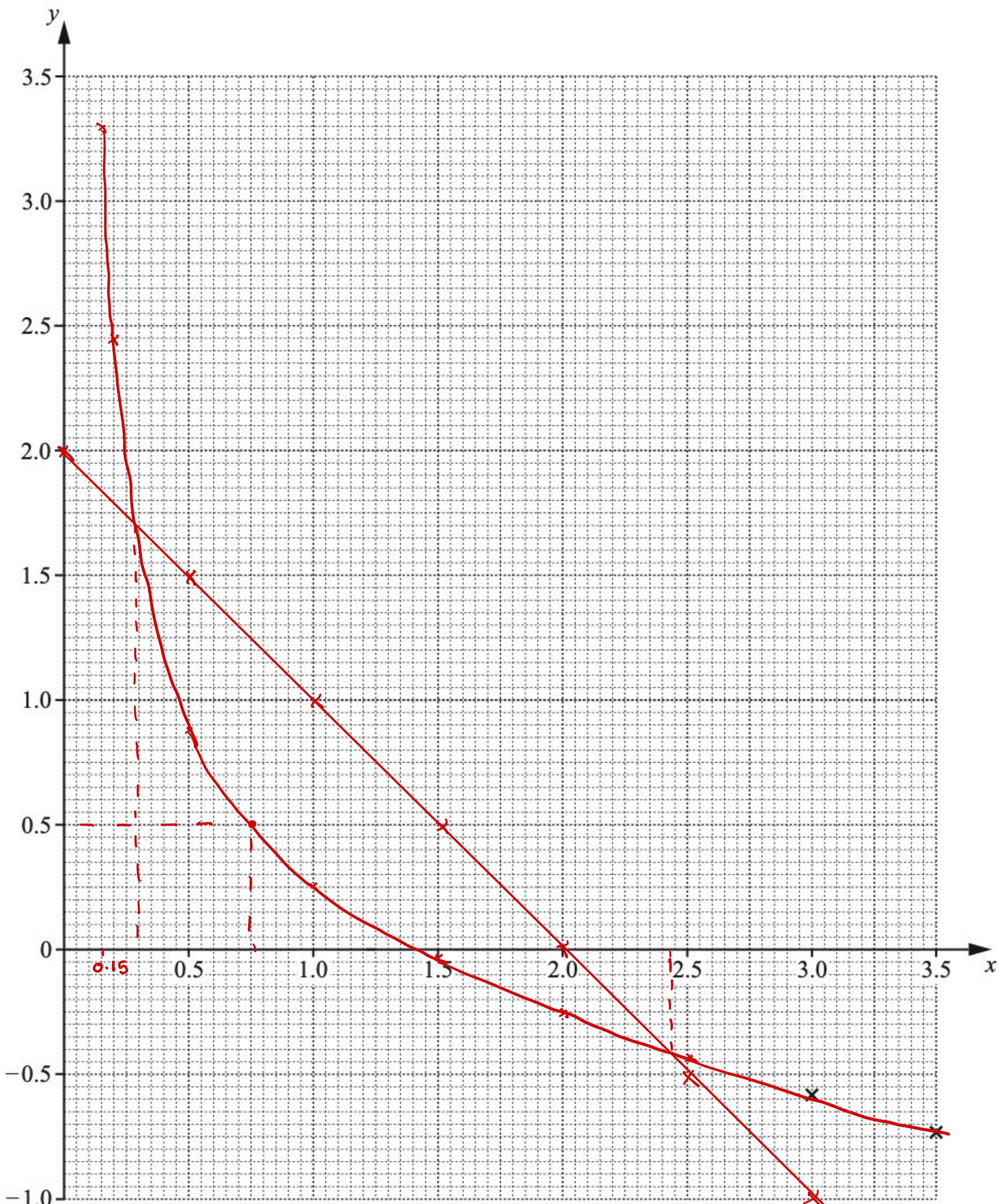
5 The table shows some values of $y = \frac{1}{2x} - \frac{x}{4}$ for $0.15 \leqslant x \leqslant 3.5$.

x	0.15	0.2	0.5	1	1.5	2	2.5	3	3.5
y	3.30	2.45	0.88	0.25	−0.04	−0.25	−0.43	−0.58	−0.73

(a) Complete the table. [3]

(b) On the grid, draw the graph of $y = \frac{1}{2x} - \frac{x}{4}$ for $0.15 \leqslant x \leqslant 3.5$.

The last two points have been plotted for you.



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[4]

(c) Use your graph to solve the equation $\frac{1}{2x} - \frac{x}{4} = \frac{1}{2}$ for $0.15 \leqslant x \leqslant 3.5$.

$x =$ 0·75 .... [1]

(d) **(i)** On the grid, draw the line $y = 2 - x$. [2]

(ii) Write down the x co-ordinates of the points where the line $y = 2 - x$ crosses the graph of $y = \frac{1}{2x} - \frac{x}{4}$ for $0.15 \leqslant x \leqslant 3.5$.

$x =$ 0·28 .... and $x =$ 2·43 .... [2]

(e) Show that the graph of $y = \frac{1}{2x} - \frac{x}{4}$ can be used to find the value of $\sqrt{2}$ for $0.15 \leqslant x \leqslant 3.5$.

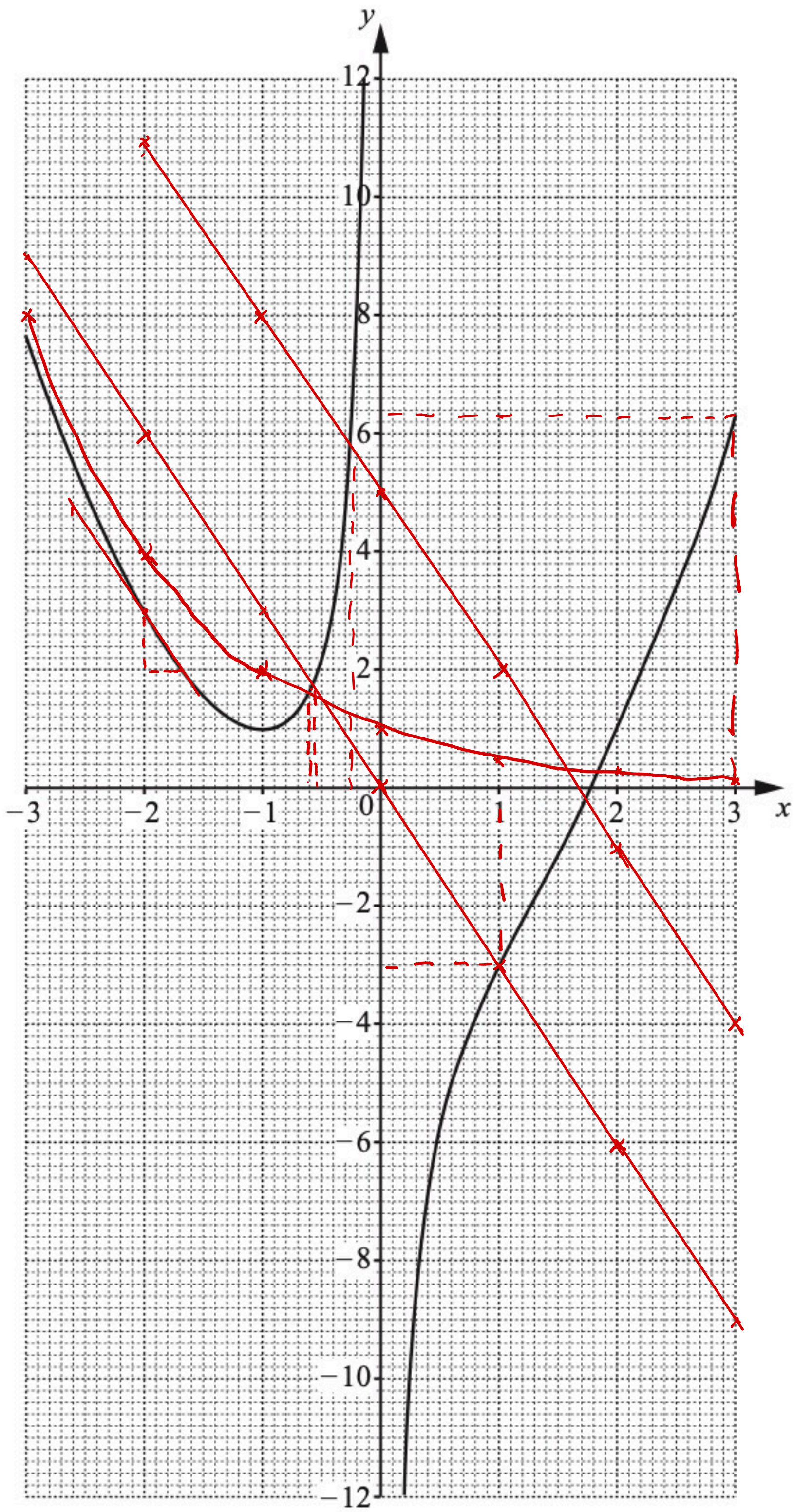
$y = \frac{1}{2\sqrt{2}} - \frac{\sqrt{2}}{4}$

$= 0$

$y = 0$, $x = 1{\cdot}42$

[2]

5 The diagram shows the graph of $y = \mathrm{f}(x)$ where $\mathrm{f}(x) = x^2 - \dfrac{2}{x} - 2,\ x \neq 0$.



(a) Use the graph to find

(i) f(1),

-3 [1]

(ii) ff(−2).

6.3 [2]

(b) On the grid opposite, draw a suitable straight line to solve the equation

$x^2 - \frac{2}{x} - 7 = -3x$ for $-3 \leqslant x \leqslant 3$.

$x^2 - \frac{2}{x} - 2 = 0$

$= -3x + 5$

$x =$ 1.74 or $x =$ -0.25 [4]

(c) By drawing a suitable tangent, find an estimate of the gradient of the curve at $x = -2$.

3.3 [3]

(d) **(i)** Complete the table for $y = g(x)$ where $g(x) = 2^{-x}$ for $-3 \leqslant x \leqslant 3$.

x	−3	−2	−1	0	1	2	3
y	8	4	2	1	0.5	0.25	0.125

[3]

(ii) On the grid opposite, draw the graph of $y = g(x)$. [3]

(iii) Use your graph to find the **positive** solution to the equation $f(x) = g(x)$.

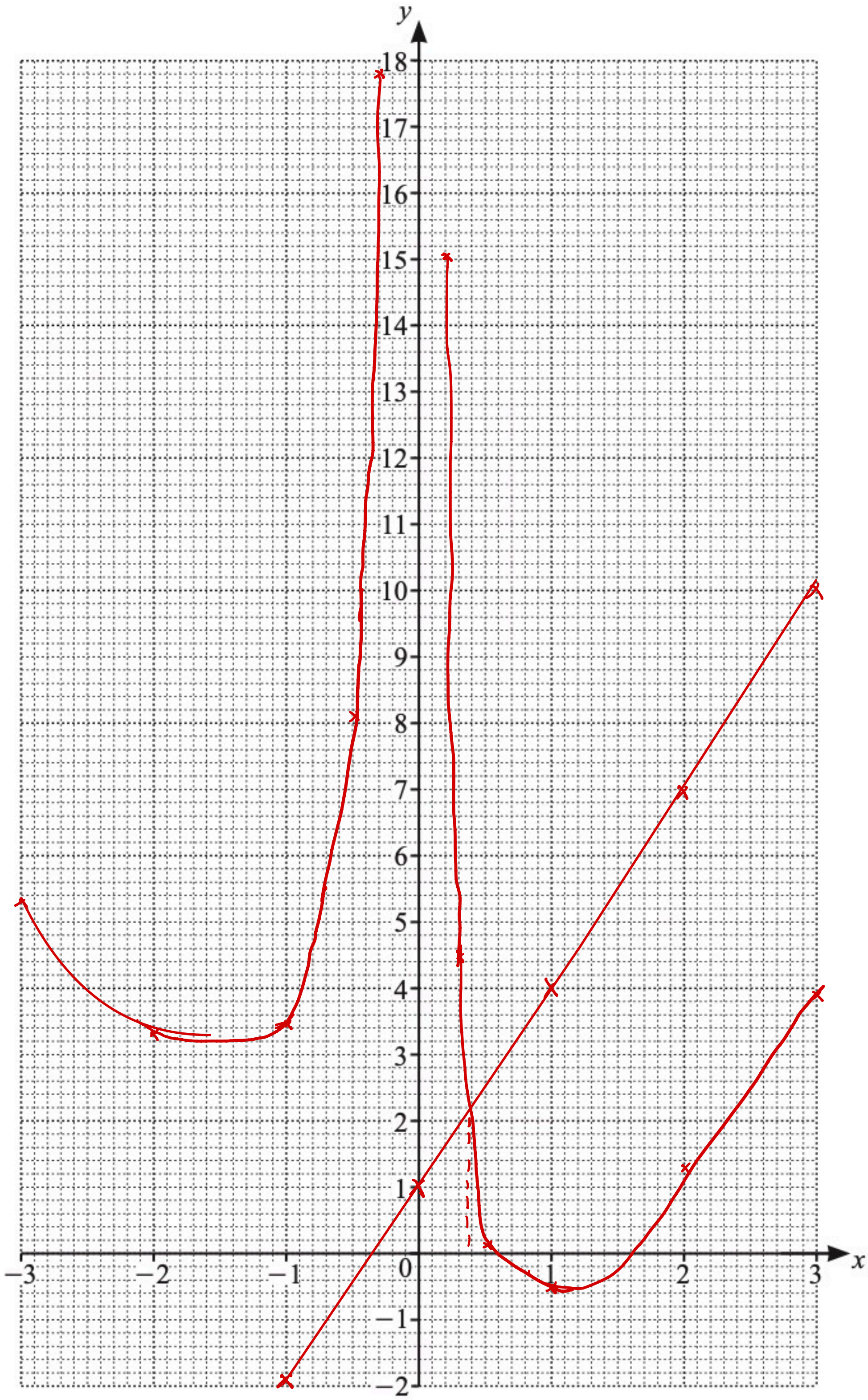
$x =$ 1.82 [1]

5 The table shows some values of $y = \frac{x^2}{2} + \frac{1}{x^2} - \frac{2}{x}$, $x \neq 0$.

x	−3	−2	−1	−0.5	−0.3		0.2	0.3	0.5	1	2	3
y	5.3	3.3	3.5	8.1	17.8		15.02	4.5	0.1	−0.5	1.3	3.9

(a) Complete the table. [3]

(b) On the grid, draw the graph of $y = \frac{x^2}{2} + \frac{1}{x^2} - \frac{2}{x}$ for $-3 \leqslant x \leqslant -0.3$ and $0.2 \leqslant x \leqslant 3$.



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[5]

(c) Use your graph to solve $\frac{x^2}{2}+\frac{1}{x^2}-\frac{2}{x}\leqslant 0$.

0.8 $\leqslant x \leqslant$ 1.6 [2]

(d) Find the smallest positive integer value of k for which $\frac{x^2}{2}+\frac{1}{x^2}-\frac{2}{x}=k$ has two solutions for $-3\leqslant x\leqslant -0.3$ and $0.2\leqslant x\leqslant 3$.

1 [1]

(e) **(i)** By drawing a suitable straight line, solve $\frac{x^2}{2}+\frac{1}{x^2}-\frac{2}{x}=3x+1$ for $-3\leqslant x\leqslant -0.3$ and $0.2\leqslant x\leqslant 3$.

$x =$ 0.38 [3]

(ii) The equation $\frac{x^2}{2}+\frac{1}{x^2}-\frac{2}{x}=3x+1$ can be written as $x^4+ax^3+bx^2+cx+2=0$.

Find the values of a, b and c.

$\frac{x^4}{2}+1-2x=3x^3+x^2$

$x^4+2-4x-6x^3-2x^2=0$

$a =$ −6

$b =$ −2

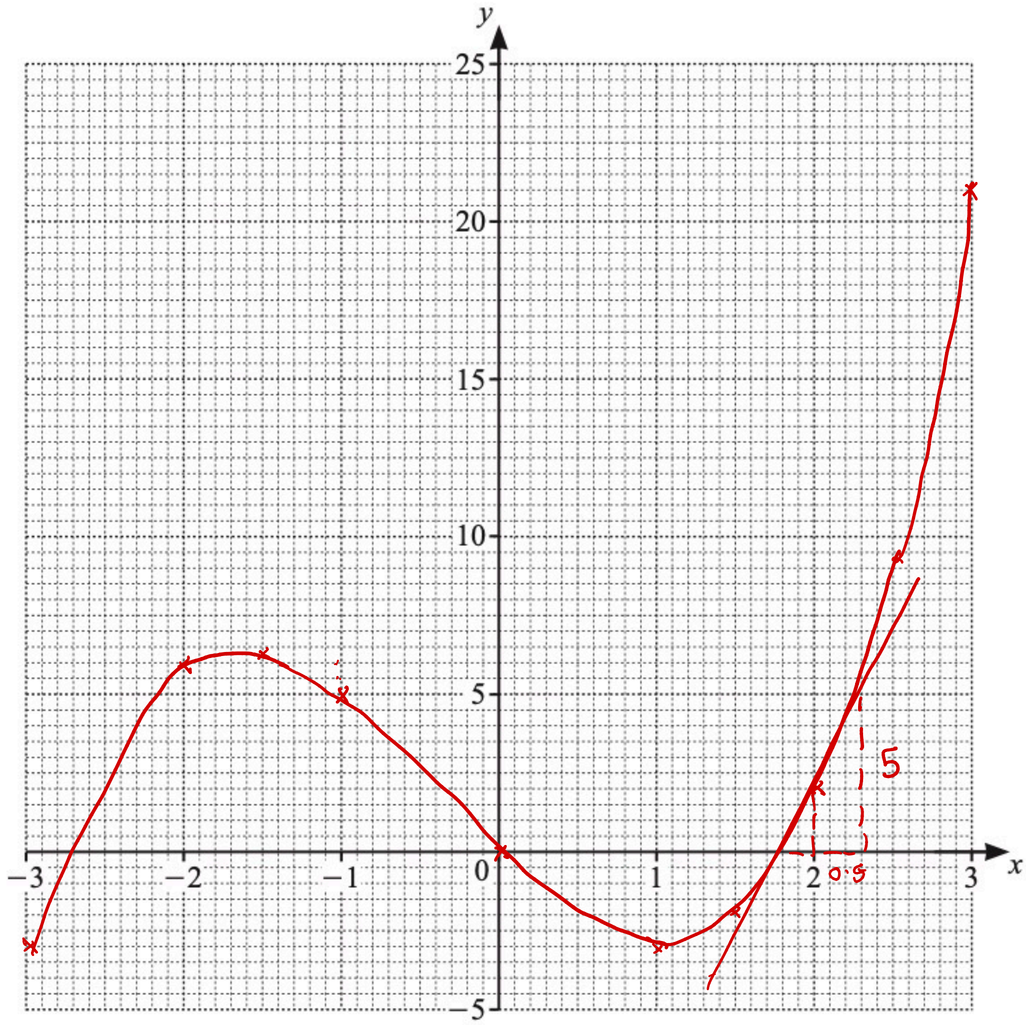
$c =$ −4 [3]

3 The table shows some values for $y = x^3 + x^2 - 5x$.

x	-3	-2	-1.5	-1	0	1	1.5	2	2.5	3
y	-3	6	6.4	5	0	-3	-1.9	2	9.4	21

(a) Complete the table. [3]

(b) On the grid, draw the graph of $y = x^3 + x^2 - 5x$ for $-3 \leqslant x \leqslant 3$.



[4]

(c) Use your graph to solve the equation $x^3 + x^2 - 5x = 0$.

$x =$ ......-2.7...... or $x =$ ......0...... or $x =$ ......1.8...... [2]

(d) By drawing a suitable tangent, find an estimate of the gradient of the curve at $x = 2$.

......10...... [3]

(e) Write down the largest value of the integer, k, so that the equation $x^3 + x^2 - 5x = k$ has three solutions for $-3 \leqslant x \leqslant 3$.

$k =$ ......6...... [1]